6. RESPONSE ANALYSIS

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6.1 General

The main purpose of a structural analysis is to synthesize a structure in such a manner that a satisfactory level of reproducing the actual response of the structure to some applied load is obtained. In undertaking such a task, engineering judgment is applied as a means to achieve a practicable solution to the physical problem at hand. This implies that restrictions are imposed on the analysis, and as a result, the accuracy of the results obtained from such analyses may accordingly also be restricted.

All constraints and limitations of an analysis are artificially imposed by the choice of the analysis method or the program which is used, or by simplifications made by the analytical engineer. Some of these constraints may be crucial to the outcome of the analysis, i.e., the predicted response.

In structural analysis, it is convenient to deal with three categories of introduced errors and inaccuracies, as follows:

- Modelling errors and inaccuracies, i.e., inaccuracies owing to idealizations and simplifications resulting from the choice of the mathematical analytical model.
- Discretisation inaccuracies, i.e., inaccuracies owing to representation of a continuum by a model with a finite discrete number of degrees of freedom.
- Numerical inaccuracies introduced in the analytical problem due to programming limitations with respect to, for example, the number of significant digits.

6.1.1 Type of Analysis

When considering the appropriate analytical method to employ, perhaps the most important initial decision is the choice of the type of analysis to be undertaken. Numerous analytical methods are currently available to the analytical engineer and may be categorized as being static or dynamic, linear or non-linear, and deterministic or stochastic. The appropriate choice of the type of analysis to be undertaken can be made as a suitable combination of these categories of analytical methods.

A diagram presenting the available types of analytical methods is shown in Figure 6.1. The various analytical methods and the derivation of these methods are discussed in more depth in the following.
6.1.2 Analytical Decisions

The various analytical options available for establishing the response of a structure in a marine loading environment are shown in Figure 6.1. Different analysis types may be most appropriate and should be chosen depending on the type of response that is required to be considered. A global strength assessment may, for example, require a different type of analysis than that required for a detailed fatigue analysis of a single node on the same structure.

Figure 6.2 and Figure 6.3 indicate typical critical analytical decisions that are required to be undertaken in assessing and obtaining the global response of a marine structure. Each of the decisions presented in Figure 6.2 and Figure 6.3 has some degree of associated uncertainty and each of these uncertainties should be evaluated individually as well as together. For example, the drag coefficients selected for an analysis may be coupled with the chosen wave theory, and the associated uncertainties will be correlated.
Dynamic response of structure dictates need for a dynamic analysis

Static Analysis

Non-linear response of structure dictates need for a non-linear analysis

Linear Analysis

Non-linear Analysis

Establish boundary condition

Yes

Non-linear formulation appropriate (λ<5D)

Potential theory based analysis (with/without Morison element inclusion)

Static or deterministic analysis

Yes

Moriason theory based analysis

Establish size of loading and structural models e.g. size of elements

Yes

Deterministic analysis

Select wave theory & method of wave/current stretching (if appropriate)

Evaluation of relevant individual sea loading characteristics, e.g.,
- water depth
- wave height, period
- current profile
- marine growth
- drag and inertia coefficients (Morison Theory)
- choice of wave phase angles

No

Stochastic or deterministic analysis

Stochastic analysis

Select appropriate modelling parameters including:
- number of wave frequencies
- choice of frequencies

Evaluation of relevant sea loading characteristics, e.g.,
- water depth
- wave spectrum data
- current profile
- marine growth
- drag and inertia coefficients (Morison Theory)

Perform analysis

Include ‘correction’ factors, as appropriate, to account for factors otherwise not considered in the analysis, e.g. dynamic effects

Analyse results

General increasing degree of complexity

Figure 6.2 Overview of Main Decisions for a Typical Global Response Analysis of a Marine Structure

Structure - Static Analysis
Guideline for Offshore Structural Reliability Analysis - General
DNV Report No. 95-2018

Chapter 6

“Guideline for Offshore Structural Reliability Analysis-General”, DNV:95-2018
Figure 6.3 Overview of Main Decisions for a Typical Global Response Analysis of a Marine Structure - Dynamic Analysis
6.2 Static Analysis

6.2.1 Linear Systems

For a static analysis the general equilibrium equation may be written as:

\[ K \mathbf{r} = \mathbf{R} \]  \hspace{1cm} (6.1)

where \( K \) is the global stiffness matrix formed from the appropriate combination of the element stiffness matrices, \( \mathbf{r} \) is the vector of the unknown nodal displacements, and \( \mathbf{R} \) is the nodal load vector.

A typical static finite element analysis based upon the above general equilibrium equation consists of the following phases, see Bell et al. (1973):

- Discretization, where the true structure is approximated by an assembly of finite, interconnected elements.
- Element analysis, where the stiffness properties of each individual element are determined and any loading is transformed into equivalent concentrated nodal forces.
- System analysis, where the merging of the individual elements of the structure is undertaken to form the element stiffness matrix and load vector, \( K \) and \( \mathbf{R} \), respectively. The nodal displacement vector \( \mathbf{r} \) (see Eq. (6.1) above) is then solved.
- Determination of elemental stresses.

6.2.2 Non-Linear Systems

Methods of solution to account for non-linearity can generally be divided into analytical (e.g., the Ritz method) and numerical methods. The numerical methods are perhaps the most prominent and are based upon the principle of stepwise integration of the problem such that any non-linear structural problem can, in principle, be solved iteratively through the solution of a series of linear problems.

When evaluating a non-linear structural system, the analytical problem reduces to that of finding the displacement vector \( \mathbf{r}(t) \) that produces an internal reaction force vector \( \mathbf{F}_{\text{int}}(t) \) that balances the applied forces \( \mathbf{R}(t) \), hence

\[ \mathbf{F}_{\text{int}}(t) \mathbf{r}(t) = \mathbf{R}(t) \]  \hspace{1cm} (6.2)

For non-linear analysis, the equilibrium equations are normally solved incrementally with corrective iterations for values of the load parameter.
6.3 Dynamic Analysis

Dynamic analyses differ from static analyses due to the inclusion of the following local and system effects:

(i) Time dependency
(ii) Damping
(iii) Inertia

In general terms, the dynamic equilibrium equation may be written as (DNV Sesam, 1993)

\[ M \dddot{r} + C \ddot{r} + K r = R(t, \dot{r}, \ddot{r}) \]  \hspace{1cm} (6.3)

where \( R(t) \) is now the time-dependent load, and \( r \) is the displacement with its time derivatives \( \dot{r} \) and \( \ddot{r} \). \( M \) is the global mass matrix, \( C \) is the global damping matrix, and \( K \) is the global stiffness matrix.

Free undamped vibration is obtained by \( C=0 \) and \( R(t, \dot{r}, \ddot{r}) = 0 \). With harmonic vibration, where \( r = \Phi \sin \omega t \), Eq. (6.3) reduces to \( (K - \omega^2 M) \Phi = 0 \).

The mass and damping properties of the system may be determined individually for each element in the system and then assembled in the same manner as described for the stiffness in Section 6.2.1. The internal reaction forces for an arbitrary element as well as the element loads may then be computed by use of virtual work equations.

Eq. (6.3) may be utilized to solve both linear and non-linear systems based upon the general principles described in Section 6.1 for static systems.
6.4 Deterministic Processes

In the context of marine structural analysis, a deterministic process is a process for which it is possible to describe the exact magnitude of the load at any given time.

Deterministic structural systems exposed to a deterministic loading will always react with a deterministic response.

For marine structures, a deterministic analysis involves an initial consideration of the statistical data representative for the environmental loading. For extreme response analysis, for example, a suitable event could be defined as the wave which is expected to produce the most severe response. This requires that the structural model is exposed to a unidirectional, periodic wave. The loading is calculated in the time domain at given points in time during a wave cycle. See Figure 6.4 below.

![Figure 6.4 Typical Simulation of a Deterministic Sea State (Time Instant)](image-url)

6.5 Stochastic Processes

A stochastic process is described by the use of probabilistics.

A stochastic load or response may not be fully described by exact magnitude at a given time, but
rather by the probability by which it will exceed some specified value.

For the simulation of stochastic wave kinematics, basically two simulation techniques are
available, namely
- digital filtering of white noise by a predicted filter
- wave superposition method.

By the digital filter method, random wave time series are generated at an origin or source point
on the surface of the water body. As the individual sinusoidal components are unknown, it is
difficult to propagate the waves to remote locations separated from the source point. Therefore
the most commonly applied simulation technique utilized is that of linear superposition of partial
waves. Several variants are available, see Langen (1981), such as
- simulation with random phase and regularly spaced frequencies
- simulation with random phase and frequency
- simulation with random phase, frequency and direction

These techniques require linear (Airy) wave theory to be utilized, see Figure 6.5. Fast Fourier
Transform (FFT) apply regularly spaced wave component frequencies. FFT is a quick way of
superposition which corresponds to the first of the above-listed simulation techniques.

Superposition of the harmonic wave components gives the following approximation to the
surface elevation $\eta$ of the irregular sea

$$\eta(r,t) = \sum_{j=1}^{n} \sum_{k=1}^{m} A_{jk} \cos(\omega_k t - k_k x + \delta_{jk})$$  (6.4)

where
- $A_{jk}$ is the amplitude for wave direction $j$ and harmonic component $k$
- $\omega_k$ is the angular frequency of harmonic component $k$
- $k_k$ is the wave number corresponding to $\omega_k$
- $\delta_{jk}$ is the phase angle uniformly distributed between 0 and $2\pi$

Fluid velocities and accelerations are given by similar expressions obtained by time
differentiations, see Sarpkaya and Isaacson (1981).

Figure 6.5 gives an illustration of the fundamental (or Central Limit) theorem, applied to
superposition of partial waves. The arbitrary surface elevation of a single sine wave has the arc
sine distribution, which is a symmetric, two-peaked distribution. By adding together two regular
waves with different frequencies, as shown in the figure, the resulting wave gets a one-peaked
distribution. By adding more and more partial waves to the sum, the resulting distribution of the
arbitrary surface elevation becomes more and more Gaussian, as apparent in the lower drawings
in Figure 6.5. It is important to include a sufficient number of equally spaced frequencies in the
sum, however, the necessary number of frequencies to be included is much dependent on the
application at hand.
6.6 Analytical Parameters

6.6.1 General

This section provides information related to choices for analytical parameters. It should be borne in mind that, if structural response resulting from any given analysis is to be evaluated against any given design criteria, standard, or code, then the undertaken analysis is to be consistent with the intention of the safety implicit in such design criteria, standard, or code.

The various options available for the choice of an appropriate analytical method are shown in Figure 6.1. There are many publications that deal with the choice of analytical procedures and give recommendations in this respect, e.g., SNAME (1993), Barltrop and Adams (1991), and Faulkner et al. (1990). In all cases, the important consideration is that all relevant global and local effects, dynamic as well as non-linear, are satisfactorily accounted for in the analysis. Perhaps the simplest method of documenting these choices is as indicated in Figure 6.2 and Figure 6.3.

Figure 6.5 Irregular Sea, Time Series of Surface Elevation, from Gran (1992)
Many different analysis models are usually available and can be applied with different degrees of simplifications. In a study by SNAME (1993), results of 16 different analyses of exactly the same structure and exactly the same loading condition were compared with respect to maximum base shear and show significant variability owing to the different analysis methods used. Figure 6.6 shows the resulting scatter in the results for the sought-after maximum base shear, City University (1993). In particular, it illustrates the impact of the inclusion of dynamics in the analysis.

**Key to Type of Analysis**
- Analytical Results 1 to 3: Single-degree-of-freedom, non-linear analysis
- Analytical Results 4 to 5: Regular wave (deterministic), linear analysis
- Analytical Results 6 to 8: Regular wave (deterministic), non-linear analysis
- Analytical Results 9 to 11: Random wave (stochastic), linear analysis
- Analytical Results 12 to 17: Random wave (stochastic), non-linear analysis

**Figure 6.6** Illustration of the scatter of a response quantity predicted by different analysis methods

The following four methods of analysis are typically used for analysis of marine structures, see Barltrop and Adams (1991)

- Design wave analysis with or without a dynamic amplification factor
- Design wave with full dynamic response analysis
- Linear stochastic, dynamic analysis (frequency domain)
- Non-linear stochastic, dynamic analysis (time domain).

Engineering simplifications may be acceptable, but the relevance of the simplified model should always be assessed in each single case.

### 6.6.2 Type of Analysis

When an analysis of a marine structure is to be undertaken, a choice must be made between a static and a dynamic analysis approach, see Figure 6.2 and Figure 6.3. Recommendations for this choice vary, however, the following guidance may be given:

- For typical fixed offshore structures (e.g., jacket type structures) the effects of dynamics, for extreme global response analysis, should be included when the global natural period of the structure is greater than three seconds.
- For floating structures, including compliant structures, a dynamic analysis should always be undertaken to identify extreme response dynamic load contributions.
- A dynamic analysis should be undertaken to establish response when impulse ('ringing') or resonant ('springing') effects may be governing.
- Model and/or prototype measurements should be considered for structures or effects not amenable to analytical calculations.

There are many different approaches that may be considered in order to account for dynamic effects. These effects may be accounted for directly by dynamic analysis, or by applying ‘correction factors’ to the results of a static analysis, see Figure 6.2 and Figure 6.3, such that the analysis may be referred to as ‘quasi-static’.

For a typical fixed offshore structure the various modeling combinations associated with the method of the dynamic loading contribution are presented in Table 6.1, see SNAME (1993).

Several topics are important and need to be addressed whenever a structural analysis is to be performed. The most important of them are listed below.

- Initial conditions and boundary conditions, such as the degree of fixity of the structure relative to its support, are sometimes critical to the sought-after response quantity predictions and should therefore be carefully assessed. Reference is made to Karunakaran (1993).
- Structural and hydrodynamic damping contributing to the resistance term in Eq. (6.3) may be critical to the result, especially in the vicinity of the natural period of the structural system.
- Linearization in frequency domain analyses is normally critical to the accuracy of the results (especially with respect to motion response in the vicinity of the natural resonance period of the structure) and must be very carefully made, see Skjong and Madsen (1987). For linearization of loads, reference is made to Section 5.1.4.3.
- The integration time step in time domain analyses.
- Transient effects in simulated load and response records should be removed.
The length of a simulated record, also known as the recording time, is related to the chosen time step length and should be appropriately chosen. Several instructions on how to harmonize simulation (or analysis) length, time step length, and transients are available, see Gran (1992), and one of them is given here:

1. Make a judgment of the highest frequency component expected in the signal. Increase the value by 20% for transient effects and hence determine the limiting frequency \( \omega_h \).
2. Choose the sampling period equal to one half the minimum required by the sampling theorem, i.e., \( \Delta t = 0.5 \pi / \omega_h \).
3. Fix the desired frequency resolution \( \Delta \omega \) or the number of points wanted in the spectrum, i.e., \( M = \omega_h / \Delta \omega \).
4. Choose the recording time as 5 times that required by the uncertainty relationship, i.e., \( T = 5 \cdot 2 \pi / \Delta \omega \). The number of sample points is then \( N = T / \Delta t \).

The importance of proper choices for simulation lengths and time step lengths is illustrated in Figure 6.7 and Figure 6.8 which are extracted from Dedden et al. (1991). Figure 6.7 shows the relative variation in maximum wave-crest elevation obtained by thirty one-hour simulations of the same wave spectrum, whereas Figure 6.8 shows the resulting thirty extreme responses of global displacement from the one-hour records presented in Figure 6.7. For the particular example considered, these figures demonstrate that for one-hour simulations the resulting simulated wave statistics from an unqualified seastate simulation may vary considerably, and the corresponding resulting characteristic response will vary even more.

<table>
<thead>
<tr>
<th>Analysis Level</th>
<th>Structural Model</th>
<th>Model of the Environmental Excitation (nonlinear)</th>
<th>Random</th>
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<td></td>
<td></td>
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<td>SDOF Linear</td>
<td>A</td>
<td>B</td>
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<td>Full results not directly available</td>
<td>Nonlinear Unsuitable</td>
<td>Unsuitable</td>
<td>Unsuitable</td>
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<tr>
<td>Complex</td>
<td>MDOF Linear</td>
<td>Generally not recommended</td>
<td>C</td>
</tr>
<tr>
<td>Full results available</td>
<td>Nonlinear</td>
<td>Generally not recommended</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Key:
- **SDOF**: Single degree of freedom model
- **MDOF**: Multi degree of freedom model
- **A**: Combines a simplified model of the structural system with a simplified model of the excitation.
- **B**: Is a refinement of Case A. It remains simple to execute and lends itself to both frequency and time domain methods. In the latter method the main nonlinearities in the excitation can be retained and therefore non-Gaussian effects in the random response can be accounted for. (Same limitations as for Case A.)
- **C**: Is a simplification of Case D, if linear modeling of the structural system is a sufficiently accurate representation.
- **D**: Is the most complete and accurate representation of reality, but also the most complex. This is a necessary combination for a detailed evaluation of the dynamic behavior of a dynamically responsive offshore structure. Both random time and frequency domain methods can be used, however, the latter requires some approximation (linearisation) of the nonlinear terms and therefore the former is the most suitable.

Table 6.1 Recommended Combinations of the Structural System and Environmental Excitation Models for a Dynamic Analysis

The statistical data associated with the information in Figure 6.7 and Figure 6.8 are included as Table 6.2 and Table 6.3, respectively, see Dedden et al. (1991).

**Figure 6.7** Variation in simulated wave crest elevation for 30 one-hour seastates
Figure 6.8 Variation in Resultant Maximum Displacement from 30 one-hour Seastate Simulations

Table 6.2 Statistical data associated with the simulated seastates presented in Figure 6.7

<table>
<thead>
<tr>
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<th>standard deviation</th>
<th>skewness</th>
<th>kurtosis</th>
<th>observed maximum</th>
<th>observed minimum</th>
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units: m, rad, s

Table 6.3 Statistical data associated with the maximum displacement response presented in Figure 6.8

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</table>

Extreme responses in terms of the characteristic largest response are described in detail elsewhere, see Sections 5.1.3.4 and 6.7.

6.6.3 Nonlinear Effects

Nonlinearities of system response may be directly calculated by the engineering tools used to analyse the structure or may be accounted for by the inclusion of ‘correction factors’ (see Figure 6.2 and Figure 6.3) during or after the structural analysis. An example of such a ‘correction factor’ typically used in structural analysis is the nonlinear amplification factor for the lateral deflection at the center of a beam-column, owing to axial loading,

$$\psi = \frac{1}{1 - \frac{P}{P_E}} \quad (6.5)$$

where

\(\psi = \) non-linear amplification factor
\(P = \) static axial load contribution
\(P_E = \) Euler buckling load

It is sometimes difficult to distinguish between model simplification and actual non-linearities of the considered structural system. For the purposes of this section no such distinction is made. The characteristic non-linearities may then be divided into four groups, namely those attributed to geometry, material, loading, and response.

It is not possible to document the significance of all nonlinearities for all marine structures. It is neither possible to document, without a specific parametric study, the importance of these nonlinearities for any particular structure as there are so many variables and interaction effects between them that may affect the sensitivity of the response to such nonlinearities. Therefore only selected examples of possible nonlinearities are listed in the following for each of the four groups identified above. It is then up to the engineer, on a case-by-case basis, to identify the importance, relevance and sensitivity of the specific response considered to the respective nonlinearities. The list is not complete, but is intended to provide the engineer with a 'starting point' for considering which nonlinear effects may be significant to the response analysis.

**Geometric nonlinearities**

- At interfaces in the structure
  - structure/foundation interaction
  - contact ‘gap’ interfaces
- Fabrication and design tolerances

**Material nonlinearities**

- Theoretical versus actual behavior
  - Ageing effects, e.g., fatigue, creep
  - Temperature effects
  - nonhomogeneous material
  - Geometrical imperfections
  - Slenderness effects (including buckling)
  - Stress/strain relationships
  - Constructional sequences
- For concrete structures the following nonlinear material characteristics should be considered:
  - Ageing effects, e.g., shrinkage, fatigue, creepage, cracking
  - Temperature effects, e.g., through-thickness temperature differences in a subsurface oil containment cell wall
  - Nonhomogeneous material (e.g., prestressing)
  - Geometrical imperfections (e.g., fabrication tolerances)
  - Stress/strain relationships (e.g., nonlinear stress/strain relationship)
  - Constructional sequences (e.g., initial force and relaxation in prestressing)
  - Three-dimensional effects (e.g., Poisson effects)
- For geotechnics the following nonlinear material characteristics should be considered:
  - Method of construction
  - Group effects
  - Type of loading
  - Nonuniform foundation
  - Ageing effects, e.g., slope stability, hydraulic stability, scour, settlement and displacements.

**Loading nonlinearities**

“Guideline for Offshore Structural Reliability Analysis-General”, DNV:95-2018
- Spatial effects (e.g. wave spreading, shielding, blockage)
- Time related effects
  - instantaneous load modelling
  - relative velocities of loading and loaded media.
- Viscous (drag) loading effects
- Damping effects
- Second-order effects, e.g., second-order bending
- Temperature effects
- Wave loading considerations
  - wave theory
  - current/wave interaction
  - variable submergence of the structure
  - run-up
  - caisson effects
  - shallow-water effects
  - wave drift
  - slamming
  - vortex shedding
  - marine growth
  - relative velocities and accelerations

**Response nonlinearities**

- Three dimensional response
- Coupled response (e.g., motion)
- Second order effects (e.g., second order bending)
  - Interaction effects
  - Dynamic effects
  - local effects combined with global effects
  - interaction with eigenmodes of the structure
  - cancellation effects
- Time related effects (e.g. Inertia effects)

### 6.6.4 Load Models

Representation of environmental loadings acting on marine structural elements is generally undertaken utilizing one, or a combination, of the following methods, see Vugts (1979) and Chakrabarti (1987)

1. **User input load condition.** A load contribution is represented as a point or distributed load specifically input by the engineer.

2. **Morison formulation.** The force per unit length normal to a slender member, subjected to a fluid flow, is calculated by the formula (Sarpkaya and Isaacson, 1981)

\[
F_n(r,t) = \rho \frac{\pi D^2}{4} C_m a_n(r,t) + \frac{1}{2} \rho D C_d v_n(r,t) |v_n(r,t)|
\]  

(6. 6)
When relative motions are involved, Eq. (6.6) may be modified to reflect such motions in the normal acceleration and velocity components $a_n(r,t)$ and $v_n(r,t)$. Further when significant, Eq. (6.6) should include both the Froude-Krylov and the wave damping forces in the formulation of $F_n(r,t)$, see Sarpkaya and Isaacson (1981). See also Chapter 5.

3. Potential theory (e.g. Radiation-diffraction analysis). This theory is based upon the assumption of potential flow of an incompressible fluid, subject to boundary conditions on the structure and a radiation condition at infinity. The solution to the potential problem is found by solving Laplace’s equation (Newman, 1977):

$$\nabla^2 \phi = 0 \quad (6.7)$$

with the appropriate boundary conditions. (By distributing sources over the wetted body surface, Green’s theorem may be applied and the source strengths are determined such that the relevant boundary conditions are satisfied, see Faltinsen and Michelsen (1974).)

4. Empirically based combinations of the above methods.

Modelling of environmental loading is one of the most important aspects in achieving an accurate model response. The load model should therefore always be adequate for the satisfactory representation of the environmental loading. (Additionally the model should be appropriate for the simulation of the response required to be analyzed in the postprocessing phase.) In this context it is important to model the following characteristics with due care and attention:

- Analysis with relevant (appropriate) theory, see Gudmestad (1990).
  The theory utilized for the analysis should be appropriate. In respect to analysis of marine structures, typical choices in this connection are
  - Selection of Morison formulation or potential theory
  - Selection of appropriate wave theory.
  In order to illustrate the importance of the choice of the appropriate theory, Figure 6.9 is included to indicate an example of the possible large loading variation that may be expected dependent upon the choice of wave theory. This figure also demonstrates that, even utilizing the same wave theory, and applying a different method of wave ‘stretching’ (e.g., ‘Airy 1’ contra ‘Airy 2’), significant variations in results can be produced.

- Size of elements.
  In the vicinity of the still water surface, it is especially important that structural elements subjected to wave loadings are not too large and that the characteristics of the wave kinematics in these locations are adequately represented. However, floating structures do need some minimum waterline area. With respect to wave loading, in deep water both water particle velocity ($u \equiv (\pi H/T) e^{i\delta} \cos \theta$) and acceleration ($\delta u/\delta t \equiv (2\pi^2 H/T^2) e^{i\delta} \sin \theta$) approach an exponential characteristic variation with elevation.

- Shape of elements.
  Especially in and around the fluid free surface and at sharp corners of the loaded structure, the shape of structural elements may be important.
  When modelling panels for potential theory analysis, ‘sharp’ corners on the loaded structure should have reduced mesh size in order to reduce possible errors resulting from extrapolated flow around such corners.
Key:
Airy 1 : Airy wave theory with constant kinematics above the still water line
Airy 2 : Airy wave theory with Wheeler stretching
Stokes 5th : Stokes 5th order wave theory
Dean 3rd : Dean stream function wave theory order 3
Dean 13th : Dean stream function wave theory order 13

Figure 6.9 Comparison of the Absolute Value of Global Base Shear (for a typical Jackup) as a Function of Water Depth for Differing Wave Theories

- Modelling of characteristic parameters.
  If Morison’s theory (Sarpkaya and Isaacson, 1984) is to be applied, the drag and added mass coefficients should be based upon relevant considerations of element roughness, Reynold’s number, and the Keulegan-Carpenter number, (Løseth and Korbijn, 1992). The procedure should depend on the type of analysis undertaken (e.g., deterministic or stochastic, static or dynamic) as well as previous calibrations of such analysis (e.g., model testing). For example, a drag coefficient utilized in a global deterministic analysis would be inappropriate in a dynamic slamming impact analysis, Det Norske Veritas (1991).

- Linearizing velocity.
  Small changes in the linearizing velocity may result in large variations in the resulting response from any given analysis. This is especially relevant to rigid body motion response at or near the natural frequency of the structure. Generally, one of three techniques is employed to obtain the linearising velocity, Det Norske Veritas Sesam (1992), namely
  - based upon a selected single value of the velocity
  - based upon linearisation characteristics associated with a single specified deterministic wave.
  - based upon linearisation with respect to a selected wave energy spectrum.
• Variable submergence.
  Loads resulting from frequency-domain stochastic wave load models generally extend only
  up to the still water surface. Local and global load effects above the still water surface
  (variable submergence effects) may therefore be ‘lost’ in the analysis. Similarly, slamming
  forces on members below the still water level may be missed.

• Nonlinear effects (see also Section 6.6.3).
  The importance of nonlinear effects is demonstrated by recent findings from the Joint
  response for a specified seastate, this JIP found a coefficient of variation (CoV) in response
  in the range of 30-50%. It was found that approximately 80% of the scatter of the low
  frequency motion was related to the formulation and magnitude of the damping.

### 6.6.5 Structural Models

As previously mentioned, all decisions taken in an analysis (see Figure 6.2 and Figure 6.3) have
some degree of uncertainty attached to them. Examples of some of the most important issues in
connection with structural modelling for any given analysis are dealt with below:

• Use of Several Different Element Types.
  Mixing several types of elements in one analysis and in one model should be avoided, if
  possible. This is so because the different theoretical foundation of different element types
  may imply discontinuous displacement fields and thereby give less accurate results.

• Mesh size and shape.
  Where membrane (3 d.o.f. ‘plate’ elements) or shell (6 d.o.f. ‘plate’ elements) elements are
  utilized in a model, the mesh of such elements should, as far as possible, have equal side
  lengths and should not have any ‘sharp’ angle corners. Additionally, for elements with more
  than 3 nodes, it should be ensured that all the nodes on such elements lie in the same plane.

• Element Size and Types.
  The model utilized in the analysis should be satisfactory with respect to ensuring that the
  model contains the correct type of fundamental elements, and is sufficiently detailed such
  that response can be extrapolated from the analysis with sufficient accuracy. The type, size
  and detail of the required model are dependent upon which type of response is to be
  analyzed. For example, recommendations for the size of model elements in the vicinity of a
  weld connection for a fatigue assessment analysis are given in Table 6.4, see Cramer et al.
  (1994), in which $t$ is the relevant thickness at the fatigue-sensitive location.

| **Table 6.4** Recommended Size of Elements for Fatigue Evaluation Studies |
|-------------------|-------------------|
| **Element Type**  | **Element Size**  |
| 20-noded solid    | $t \times t \times t$ |
| 8-noded shell     | $2t \times 2t$ |
| 4-noded shell     | $t \times t$ |

“Guideline for Offshore Structural Reliability Analysis - General”, DNV:95-2018
The maximum size of elements should be such that the required accuracy of the response information is satisfactorily reflected. On the other hand, if, in a linear elastic analysis, the element size, at corners of the structure, is too small the stress levels will approach infinity and the stress response, at these corners, will be relatively meaningless information.

### 6.6.6 Analysis Model

In connection with structural analytical methods, it is important to consider the following characteristics

- **Analysis with relevant (appropriate) theory.**
  The theory utilized for the analysis should be appropriate. In respect to analysis of marine structures, typical choices in this connection are
  - Static analysis may not be acceptable for a problem where dynamic loadings dominate
  - Linear analysis may not be appropriate for a structure with large displacement responses
  - The theory of extrapolation of extreme values should be appropriate

- **Solution techniques.**
  Program software allows different solution techniques to be utilized when undertaking a dynamic analysis in order to reduce CPU time, see Det Norske Veritas Sesam (1991). Some techniques are appropriate to some types of analysis whilst the same solution technique may be quite inappropriate for another type of analysis. As an example, Component Mode Synthesis, Det Norske Veritas Sesam (1991), is a technique in which the dynamic behavior of the interior of a super element is accounted for by replacing discrete degrees of freedom by eigenvectors (mode shapes). The technique is well suited for free vibration analysis and other dynamic analyses in which, primarily, the deflections are to be analyzed, however, when stresses and/or forces are sought, the Component Mode Synthesis analysis may provide inaccurate solutions. This is because stresses and forces are functions of the difference in displacements of neighboring nodes and such local information is often poorly represented by the eigenvector of the lower eigenfrequencies.

- **Poor conditioning.**
  The numerical accuracy of the equation solution is related to the ratio between the smallest and largest stiffness in the structural model. Numerical inaccuracies may result from ill conditioned models.

### 6.6.7 Evaluation of Response

In connection with the evaluation of response it is important to consider the following characteristics:

- **Resultant stresses**
  For stress-contour (iso-curve) plots, generally, nodal averaging techniques should not be utilized. Such plots may be inaccurate due to differing local axis systems at the nodal points. Elementwise averaged stress plots will provide information on the accuracy of the results, as large stress discontinuities at the boundaries of the elements will indicate that the mesh is not fine enough. For numerical presentation, Gaussian results may be the most appropriate due to the fact that these are the primary results provided by the undertaken analysis.

- **Through-thickness bending**
When evaluating plots of a structure containing elements with more than one surface (e.g. shell elements) it should be ascertained that all elements have the same surface presented on the same plot and that relevant account of the through-thickness bending in the plot has been fully considered.

- **Expectations**
  
  The response expectations resulting from an undertaken analysis should be evaluated against those responses actually resulting from the analysis. For example, responses that cannot be explained (i.e., are non-physical) should be a reason for questioning the validity of the analysis. See also comments given in Section 6.6.8 below.

### 6.6.8 Analysis Errors

Human error is perhaps the most dominant cause of inaccuracies resulting for response analysis, see Nielsen (1992). Further, the more complicated (and complete) an analytical procedure, the more likely it is to commit errors in a corresponding analysis.

In structural reliability analysis it is not possible in any meaningful way to take account of human errors. However, if we refer to relevant joint industry studies undertaken, see for example Nielsen (1992), it is clear that one major source of uncertainty in any given analysis is associated with human error. Findings from the Joint Industry Project ‘FPS 2000 Comparative Study’, Nielsen (1992), indicate that the average value for the coefficient of variation (CoV) obtained from comparison of 23 institution analysis results is approximately halved by removing those analysis results that contain obvious errors in either the modelling, or the use of appropriate theory. The effects of quality control of an analysis are clearly visualized by comparing Figure 6.10 and Figure 6.11. These figures show first order wave motion results from analyses by 23 institutions of the same ‘Deep Draught Floating Structure’ based on the same specified environmental criteria. The original results from the study, as submitted to the project by the institutions, are presented in Figure 6.10. The results were subjected to limited quality control based upon the following criteria ;

(i) Results were rejected if an inappropriate theory was utilized to develop the load model (i.e., methods other than 3D radiation/diffraction theory were rejected).

(ii) Results were rejected if an inappropriate model mesh size was utilized (i.e. excessively large ‘panel’ mesh elements).

(iii) Results were rejected if obvious non-physical characteristics were found from the presented results (e.g., damping at low frequencies did not tend to zero, added mass did not approach a constant value at high and low frequencies etc.)

After such control the results presented in Figure 6.11 were obtained. (Out of the original 23 sets of analysis for the extreme motion response no fewer than 4 sets of results were discarded based upon the above stated criteria.)
In order to reduce modelling errors, all analyses should be subject to satisfactory verification, i.e., quality control. An essential part of such control should always include a review of load sums, reaction forces, and displacement plots.
6.6.9 Statistical Properties of Response Analysis

Chapter 6 deals, in general terms, with response analysis. Figure 6.2 and Figure 6.3 are presented in order to document typical decisions that need to be undertaken in order to analyze a structure in a marine loading environment. Everyone of these decisions has associated uncertainties. As such, it is not possible to generalize in respect to providing relevant model uncertainty data. Model uncertainty statistics should therefore be resolved as appropriate to the specific analysis being considered.

The most appropriate data sources for obtaining statistics applicable to analytical (model) uncertainty may be obtained from large joint industry projects, see City University (1993), Nielsen (1992), and Taylor and Jefferys (1986). When deciding upon the statistical database to be utilised as such a source for obtaining model uncertainty the following items should, however, be considered:

- Consistent definition of model uncertainty
  Model uncertainties should be accounted for in the reliability analysis, e.g., by random factors on quantities computed by some particular model which is applied. Such a model uncertainty representation by random factors should be consistent with the model uncertainties which are included or inherent in available source data. This applies to choice of bias, coefficient of variation, and distribution type for the random model factors which are included in the reliability analysis. They must all be properly chosen. It is important to ensure that the statistical model used as a basis for obtaining estimates of model uncertainty properties from source data includes the same or similar model parameters as those included in the reliability analysis.

\[ \eta = \begin{cases} 1.843, & \sigma = 0.206, & \text{CoV} = 0.112 \\ 2.444, & \sigma = 0.441, & \text{CoV} = 0.180 \end{cases} \]

Figure 6.12 Variation in the Global Response of Displacement (Sway) Resulting from Different Types of Analysis

- Consistent analytical basis
  The type of analyses (see Section 6.1.1) utilized in obtaining model uncertainty statistics should, as far as possible, be consistent with the type of analysis undertaken as the basis for the reliability study because statistics for model uncertainty are dependent upon the type of analysis undertaken. Figure 6.12 demonstrates the variability in the Coefficient of Variation (CoV), as obtained in one Joint Industry Project, City University (1993), as a function of the type of analysis undertaken. This figure shows that, for one specific response, namely that of global rig displacement, when utilizing the same model and loading data, the CoV is found to vary from 0.11, for static analysis, to 0.18 for dynamic analysis. (Note: The dynamic linear and non-linear analysis results presented in Figure 6.12 are subsets of the dynamic analysis result set.)
  The analytical information specified by the project controllers, in connection with the data shown in Figure 6.12, City University (1993), was
  - complete information related to structural stiffness
  - complete information related to the static load condition
  - complete information related to the boundary conditions
  - Specified hydrodynamic coefficients and wave theory.

- Consistent response basis
  When considering model uncertainty based upon the findings from a specific study it should be ensured that such uncertainty is obtained from an appropriate (similar) response analysis. Reference is made to the FPS 2000 joint industry project, Nielsen (1992), in order to demonstrate this. Utilizing exactly the same model and exactly the same environmental criteria, the coefficients of variations in first-order responses for different degrees of freedom are quite different. For example, the coefficient of variation for the extreme combined response of heave is about one third of that for pitch, even though exactly the same basis
analyses have been applied for obtaining the results, see Table 6.5. Another example of such scatter is provided in Table 6.6, City University (1993).

### Table 6.5 Example of Global Response Statistics from a Comparison of the Results from Twenty Three Different Analysis of a Deep Water Floater, from Nielsen (1992)

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<td>Heave (m)</td>
<td>Pitch (deg)</td>
<td></td>
<td></td>
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<tr>
<td>First Order</td>
<td>Average</td>
<td>1.96</td>
<td>0.16</td>
<td>0.46</td>
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<tr>
<td>Response</td>
<td>Standard Deviation</td>
<td>0.255</td>
<td>0.026</td>
<td>0.147</td>
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</tr>
<tr>
<td></td>
<td>Coefficient of Variation</td>
<td>0.13</td>
<td>0.16</td>
<td>0.32</td>
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<tr>
<td>Low Frequency</td>
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<td>0.54</td>
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<tr>
<td>Response</td>
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<td>0.612</td>
<td>0.139</td>
<td>0.351</td>
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<tr>
<td></td>
<td>Coefficient of Variation</td>
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<td>0.58</td>
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<tr>
<td>Extreme Combined</td>
<td>Average</td>
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<td>0.95</td>
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<tr>
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<td>Standard Deviation</td>
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<td>0.418</td>
<td>1.503</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coefficient of Variation</td>
<td>0.26</td>
<td>0.19</td>
<td>0.52</td>
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</tr>
</tbody>
</table>

### Table 6.6 Example of Global Response Statistics from a Comparison of the Results from Sixteen Different Analyses of a Jackup, City University (1993).

<table>
<thead>
<tr>
<th></th>
<th>SWAY (m)</th>
<th>O.T.M. (MNm)</th>
<th>B.S. (MN)</th>
<th></th>
</tr>
</thead>
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<tr>
<td></td>
<td>Static</td>
<td>Dynamic</td>
<td>Static</td>
<td>Dynamic</td>
</tr>
<tr>
<td>Average</td>
<td>1.858</td>
<td>2.444</td>
<td>1014.3</td>
<td>1425.7</td>
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<tr>
<td>Standard Deviation</td>
<td>0.205</td>
<td>0.441</td>
<td>132.884</td>
<td>287.467</td>
</tr>
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<td>Coefficient of Variation</td>
<td>0.11</td>
<td>0.18</td>
<td>0.131</td>
<td>0.202</td>
</tr>
</tbody>
</table>

**Key:**
- O.T.M.: Overturning Moment
- B.S.: Base Shear
- Static: Results from static analysis
- Dynamic: Results from dynamic analysis

6.7 Stochastic Response Analysis

6.7.1 General

This section describes the response analysis and gives some guidance related to choice of method. The response analysis method must be chosen in connection with the type of information available for the loading, as described in the above section. Since there are several methods available in the literature described in various notations, a description of the methods are made. For further details references is made to Wen (1987), Ferry-Borges (1972), Bjerager et al. (1988), Ronold (1990).

First the structural analysis techniques are described, and then the statistical analysis of the results are covered. The results of all these analyses are given as the probability of failure during a reference time period $P_{F,T}$ which is the goal of the analysis. All other probabilities of failure given in this chapter are conditional probabilities that are not to be compared with the target reliabilities since the type of resistance is not covered in this chapter.

6.7.2 Long term statistics

In this analysis the response computation is performed ahead of the extreme value consideration such that the extreme load effect is determined rather than the extreme loads. The distributions for load events at a random point in time/space is applied, that is a joint environmental description as described under Chapter 5. The long-term environmental conditions are needed for this type of analyses. The amount of data required for such an analysis could be greater than for an event based analysis.

- **Long Term Statistics by Independent Sea States**

  This approach was formulated in Bjerager et al. (1988). The method is judged to be accurate in prediction of extremes. Especially in cases where several environmental loads are to be combined this method is recommended. The method also includes the cases where the failure criterion is based on a combination of load effects, such as bending and axial forces although this is not discussed in detail here. Applications are shown for different structures: a jacket, Løseth and Bjerager (1989), a moored structure, Bjerager et al. (1988), and a jack-up structure, Løseth et al. (1990). The probability of failure during a storm of duration $D$ is defined as

  $P_{F,T} \mid Y, Z = \min_{[0,D]} g(X(t) \mid Y(t) = y, Z = z) \leq 0$  \hspace{1cm} (6.8)

  where $X(t)$ contains the fast response processes, $Z$ contains the stochastic time independent variables and $Y(t)$ contains environmental processes. An illustration of time scales are shown in Figure 6. 13.
These slowly varying environmental processes are assumed to be constant during one seastate, (stationary $X(t)$), and can thus be represented by the outcome of time independent random variables $Y(t) = y$ during one seastate of duration $D$.

The vector of environmental variables may, for example, be set to $Y = [H_S, T_Z, V, U_{10}]^T$, where:

- $H_S$: Significant wave height
- $T_Z$: Zero wave crossing period
- $V$: Wind driven current velocity
- $U_{10}$: Average wind speed at 10 meter elevation

In the case that the fast process is a scalar process $X(t)$, the minimum of the limit state function over a seastate duration, see Eq. \((6.8)\), occurs when the process takes on its extreme value. A distribution function for the extreme event can in this case be computed by an analytic expression and in the following a scalar process will be assumed. The extreme during one seastate, $X_{\text{max}}$, treated by a stochastic variable describing the inherent uncertainty of the extreme, is represented approximately as
where \( \nu_r(X(t)) \) is the upcrossing rate of the resistance level \( R = r \), which may be related to the variables \( Z = z \). This result follows from the character of the point process of upcrossing of level \( r \) as being asymptotically a Poisson process for \( r \to \infty \). The upcrossing rate is treated further in Appendix D.

Since the above failure probability is determined by conditioning upon the environmental variables and the variables describing the failure surface, an unconditioning procedure is required. To utilize the efficient reliability methods in this procedure, a limit state formulation is required to carry out the integration over all \( y \) and \( z \). As described in Bjerager et al. (1988), this is done by realizing that the probability of failure can be expressed in terms of a quantile in the standard normal distribution. \( P_{F_{0}}|y, z = \Phi(u_{aux}) \), the conditional probability of failure, can then be rewritten as

\[
P_{F_{0}}|y, z = \begin{cases} 
0, & \text{if } u_{aux} < 0 \\
1, & \text{if } u_{aux} \geq 0 
\end{cases}
\]

(6.10)

where \( u_{aux} \) is a standard Gaussian auxiliary variable and \( u_{aux} \) is a realization of this variable. The integration over all possible outcomes \( y \) of the environmental variables \( Y(t) \) can now be carried out according to the law of total probability and gives

\[
P_{F_{0}}|z = \int_{\mathbb{R}^{p}} \left[ P\left[ U_{aux} - \Phi^{-1}(P_{F_{0}}|y, z) \leq 0 \right] f_{y}(y)dy \right]
\]

\[
= P\left[ U_{aux} - \Phi^{-1}(P_{F_{0}}|Y, z) \leq 0 \right] 
\]

\[
= P[h(Y, U_{aux}|z) \leq 0] 
\]

(6.11)

where \( h(Y, U_{aux}|z) \) is the limit state function for this problem, \( \mathbb{R}^{p} \) represents all possible outcomes of the vector \( Y \) with dimension \( p \) and \( f_{y}(y) \) is the probability density of \( Y \) within a random seastate, i.e., "the long term" distribution. The problem has now been reformulated in a way that allows solution by reliability methods such as FORM and SORM. For simpler notation, the variable \( U_{aux} \) is assumed to be included in the vector of environmental variables, as \( Y = (H_{5}, T_{s}, V, U_{10}, U_{aux})^{T} \). \( U_{aux} \) will here represent the inherent uncertainty of the extreme of the fast process. A limit state function for load capacity can in this case be written as

\[
h(Y|z) = R(z) - X_{\max,0}(Y, z) 
\]

(6.12)

in which \( R(z) \) is a deterministic function of \( z \).
In a time period $T$, longer than the duration of one storm $D$, there will be a number of outcomes of the process $Y(t)$. By assuming that the events in each sea state are independent of each other, the probability of failure during a period of time $T$ is given by

$$P_{f_T|Z} = 1 - (1 - P_{f_D|Z})^{N_{sea}} \quad (6.13)$$

where the number of storms is determined as $N_{sea} = T / D$.

Finally, the condition $Z = z$ is eliminated by applying the law of total probability once more and integrating over all $Z$, and the probability of failure during the period of time $T$ becomes

$$P_{f_T} = \int_{Z \in \mathbb{R}^q} \left[ P_{f_T|Z} f_Z(z) dz = P\left[ V_{aux} - \Phi^{-1}(P_{f_T|Z}) \leq 0 \right] \right. \quad (6.14)$$

where $\mathbb{R}^q$ represents all possible outcomes of the vector $Z$ with dimension $q$ and $f_Z(z)$ is the probability density of $Z$. Also the solution of this integral may be obtained by reliability methods, as indicated by the right hand side of Eq. (6.14), introducing another auxiliary variable $V_{aux}$ which is analogous to $U_{aux}$.

The result of entire procedure can be expressed as

$$1 - P_{f_T} = \int_{Z \in \mathbb{R}^q} \int_{Y \in \mathbb{R}^p} \left\{ 1 - P_{f_D|Y}(X(t)|Y, Z) f_Y(y) dy \right\}^{N_{sea}} f_Z(z) dz \quad (6.15)$$

The method uses a number of independent events $N_{sea}$ which should be calibrated against the scale of fluctuation of the environmental variables. If such a calibration is not performed, it is recommended to use a duration $D=20$ minutes which is a common recording period for seastate characteristics. The estimates of reliability are more conservative for short storm durations than for long storm durations.

In the above method the environmental variables are described by continuous joint distributions. The failure criterion should be described during stationary sea states of given durations. The advantage of the method is that several combined environmental processes can be treated by efficient reliability methods. The method will also detect if there are other than the extreme sea-states that contribute to the failure probability. It can also be used in conjunction with vector processes, e.g., for combination of axial, shear and moment stresses within the failure criterion, as described in Appendix D.

**Long-Term Statistics by Independent Amplitudes**

This approach is similar to the above, but the amplitudes of the sea elevation process are analyzed instead of the process itself. For further reference see Battjes (1972), Nordenstrøm (1973), Ingles et al. (1985), and Mathisen (1991).

The probability of failure for one amplitude is established by
1 − \( P_{F,a}|z \) = \[ \int_{h_s}^{h_f} \int_{t_z}^{t_f} \frac{V_0(h,t_z)}{V_0} \left( \frac{1}{\sqrt{2\pi\sigma}} \right)^{-\frac{1}{2}} \exp\left( -\frac{a^2}{2\sigma^2} \right) \] \[ F_{A}(a|h_s,t_z,z)f_{H_s}(h_s,t_z)dt_z dh_s \] \hspace{1cm} (6.16)

where \( \frac{V_0}{V_0} \) is the average zero-upcrossing rate integrated over all possible outcomes of the significant wave height \( H_s \) and the zero-upcrossing period \( T_z \). The distribution amplitudes may be estimated from a Rayleigh distribution or a Rice distribution, when the sea elevation process is represented by a Gaussian model. For a non-Gaussian model, some application of a Hermite model may be used, see Winterstein and Ness (1989). The parameters of the distribution of amplitudes are dependent on \( H_s \) and \( T_z \). For the narrow-banded Gaussian case, the Rayleigh distribution is appropriate and is given by

\[ F_{A}(a|H_s,T_z,z) = 1 - \exp\left( -0.5 \frac{(a - \mu_a)^2}{\lambda_0} \right) \] \hspace{1cm} (6.17)

Where \( \lambda_0 \) is the 0th spectral moment defined as a function of the wave spectrum which in turn is defined in terms of \( H_s \) and \( T_z \). The failure probability during the period \( T \) may be determined by assuming that each of the response amplitudes are independent, hence

\[ 1 - P_{F,a}|z = (1 - P_{F,a}|z)^{N_a} \] \hspace{1cm} (6.18)

where \( N_a = T \frac{V_0}{V_0} \). For further integration over the time-independent variables, consult the previous explanation on the long-term statistics.

As an alternative, the extreme response \( X_{max}(z) \) may be determined by the assumption that the extreme value distribution has a small scatter,

\[ \frac{1}{N_a} = P_{F,a}|z \] \hspace{1cm} (6.19)

thus giving an approximate value for the most probable extreme value by solving for the corresponding amplitude \( a = X_{max}(z) \). This will ignore the scatter of the extreme value.

The determined extreme amplitude is to be implemented in the limit state function for integration over the time independent variables, such that the total probability of failure during the period is determined by

\[ 1 - P_{F,T} = \int_{z \in \mathbb{R}^n} (1 - P[h(Z) \leq 0]) f_z(z) dz \] \hspace{1cm} (6.20)

where the limit state to be used in the integration may be written as

\[ h(Z) = R(Z) - X_{max}(Z) \] \hspace{1cm} (6.21)
in which $R(Z)$ is a function of $Z$. Note that by this method the amplitude is averaged by use of Eq. (6.16) over storm parameters which by themselves are averages over a storm duration $D$. The result is raised to a power equal to the number of amplitudes, which is greater than the number of storm durations, thus giving a conservative estimate.

This method does, however, give quite similar results to “the long term statistics by independent sea states”, as indicated in Løseth and Bjerager (1989) and demonstrated by Mathisen (1991).

This method is applicable only where the environmental variables are to be treated. The integration over seastate variables cannot be directly performed by reliability methods without simplifications, such that a numerical integration should preferably be applied. This introduces a limitation on the number of environmental variables that can be treated. It is also difficult to combine this method with time independent stochastic variables. This approach is, however, established from past applications on marine structures, and is conveniently applied in conjunction with estimation of fatigue damage.

6.7.3 Extreme Environmental Events Statistics

Here the methods are based on determining the extreme load events in advance of the response analysis. The basic extreme value consideration is then made ahead of the response analysis, although there may also be an extreme value consideration to be made within the event, e.g., if the event is defined as a storm of duration $D$.

Examples of analyses are:

- A single wave load analysis using extreme wave height and extreme current.
- A specified summer storm condition for summer operation.
- A pre-specified explosion or fire heat.
- Design against a specified tank pressure.
- Ship collision.

In reliability terms such an analysis is performed by conditioning on the extreme event. For environmental variables the distributions for the extreme loads are determined from Chapter 5, under short term environmental conditions and instantaneous design events. The combination of load effects has to be performed using a load combination rule as discussed below. The maximum of all loads together will usually give conservative results, if these do not act in opposite directions or in similar ways cancel each other. The methods described in this chapter are all conditional on certain events, and therefore it should be documented that an event considered is actually an event that governs the design.

The reason for conditioning on the occurrence of, e.g., an extreme significant wave height $H_{\text{max}}$ or a single wave amplitude $\eta_{\text{max}}$, is that this gives a simpler formulation and a numerically more stable procedure as it avoids the nested application of the reliability methods as described in Section 6.7.2. Since the most important variable is usually known in advance to be the wave height, the procedure will give good results. The use of the method may, however, introduce pitfalls, through the conditioning, if there are other important variables that are only introduced by conditional distributions. Such a situation might arise if a dynamically sensitive structure is analysed conditional on a particular extreme wave height.
• **Extreme Seastate**

In many offshore cases it turns out that the significant wave height is by far the most important environmental variable. In that case the extreme load effect can be approximated by conditioning on the extreme significant wave height \( H_{\text{max}} \).

The probability of failure during the reference period \( T \) can be determined by the integration of the failure probability over the outcomes of the stochastic variables as

\[
P_{f_{ij}}(H_{\text{max}} = \int_{x \in \mathbb{R}^n} \int_{y \in \mathbb{R}^n} (F_{f_{ij}}(y^*, z)f_Z(z)dy^*dz)
\]

where \( Y^* \) equals the vector \( Y \) except that it now contains the extreme significant wave height instead of the arbitrary significant wave height, hence \( Y^* = [H_{\text{max}}, T_z, V, U_{10}, U_{\text{aux}}] \). The distribution of the extreme significant wave height \( H_{\text{max}} \) has the distribution derived from the “long term” distribution of the arbitrary significant wave height as

\[
F_{H_{\text{max}}}(h) = \left[ F_{H_{ij}}(h) \right]^{N_{\text{aux}}}
\]

A limit state function for load capacity, to be used for solution of this integral, can in this case be written as

\[
h(Y^*, Z) = R(Z) - X_{\text{max},p}(Y^*, Z)
\]

where \( R \) is some function of \( Z \).

This method can become non-conservative in cases where the other environmental variables are important to the result, e.g., the peak period or the current. This method should be considered only the significant wave height is the dominating variable. If the significant wave height is not the only dominating environmental variable, the section on "combination of events" should be consulted. Some long term method may be considered as an attractive alternative, because nested applications of reliability methods can then be avoided, and a more stable numerical procedure can thus be established.

• **Extreme Wave Height**

In case the wave process is the most important for determining the environmental load, the extreme wave height may be used to compute the environmental loads on the structure. From the vector \( Y^* \) the extreme wave height during one storm and the corresponding period have to be determined. The vector of environmental variables is then to be expanded into \( Y^{**} \) which contains \( Y^* \) but has one additional variable to account for the individual period of the extreme wave, \( T_W \). The variable \( U_{\text{aux}} \) here represents the inherent uncertainty of the surface
elevation process. The vector of environmental variables is in this case
\[ Y^{*} = \begin{bmatrix} H_{s,aux}^*, T_z, V, U_{10}, U_{aux}, T_w \end{bmatrix}^T. \]

Under a Poissonian assumption for rare events, the distribution of a large wave amplitude \( \eta \) of a Gaussian surface elevation process can be written as
\[ \Phi(U_{aux}) = F_q(\eta | h, t_c) = \exp(-D\nu_q) \] (6.25)

where
\[ \nu_q = v_0 \exp(-\frac{\eta^2}{2\lambda_0^2}) = \frac{1}{2\pi} \left( \frac{\lambda_0}{\lambda_0^2} \right)^{\frac{1}{2}} \] (6.26)

Solving for the amplitude of the individual wave, this gives
\[ \eta_{max} = -2\lambda_0 \ln \left[ \frac{2\pi \left( \frac{\lambda_0}{\lambda_z} \right)^\frac{1}{2}}{D \left( -\ln \Phi(U_{aux}) \right)} \right]^{\frac{1}{2}} \]
\[ \lambda_0 = \frac{H_s^2}{16} \quad \lambda_z = \frac{H_s^2}{16} \left( \frac{2\pi}{T_z} \right)^2 \] (6.27)

A correction for bandwidth may be introduced as shown in Madsen et al. (1986), Example 5.4, and when the surface elevation is non-Gaussian, the upcrossing rate has to be determined, e.g., using the Hermite model, Winterstein (1985), as shown in Bjerager et al. (1988).

For the corresponding wave period a suitable distribution may be derived from the Longueto-Higgins joint distribution for wave height and period as
\[ F_q(\eta | H_{s,aux}, T_z, \eta) = \frac{\Phi \left( \frac{\eta \lambda_1}{\sqrt{\lambda_0 \lambda_2 - \lambda_1^2}} \left( 1 - \frac{\lambda_0}{\lambda_1} \right) \right)}{\Phi \left( \frac{\eta \lambda_1}{\sqrt{\lambda_0 \lambda_2 - \lambda_1^2}} \right)} \] (6.28)

The probability of failure during the reference period is in this case evaluated as
\[ P_{fr} = \int_{\mathbb{R}^n} \int_{\mathbb{R}^p} (P_{y^*, z} | y^*, z) f_{y^*}(y^*) f_z(z) dy^* dz \] (6.29)

This probability of failure may be determined by use of reliability methods directly, without nesting. The method may, however, be the only available path to arrive at a failure condition in a practical case, because of computational difficulties, failure formulations, etc. This method should be considered if the wave height is the dominating variable and, if not, then the considerations made below for combination of events should be made.
• **Other Types of Events**

If other events such as explosions or collisions are to be used as the conditioning event in a reliability analysis, the procedure is similar to that of the conditional extreme wave analysis. The probability of failure given the event would be determined as

\[
P_{F|e} = P\left[G(X^*|Z^* = z^*) \leq 0 \right] \tag{6.30}
\]

where \(X^*\) are the variables related to the "within event" uncertainties and \(Z^*\) are the variables that are constant over time. These could be related to the initiation of the considered extreme event.

Further an integration over the possible time independent variables should be performed.

\[
P_{F,T} = \int_{z \in \mathbb{R}^t} P_{F|e} |z^* f_{z^*}(z^*) dz^* \tag{6.31}
\]

This analysis is comparable to a risk analysis concept where those variables in \(Z\) describing the initiation of events have a discretized probability of occurrence \(f_{z^*}(z^*) dz^* = P_i\) as in a fault or event tree.

### 6.8 Load and Response Combinations

In the previous sections, the time dependence has, in cases of a single time-dependent variable, been treated by transforming to a variable with a distribution for the largest maximum during a time period. In the cases where a crossing analysis is applicable, as described in Appendix D, combination of simultaneous processes may be performed, and application of the long term method described in Section 6.7 also combines the environmental parameter variations over intervals in time. For the methods that use conditioning on an extreme event, and where time varying loads act in combination on a structure, some approximate method is required for the combination of the load effects. The combination is approximate since a transformation to one single variable has been performed in order to arrive at the extreme event that describe the largest maximum only, and therefore the detailed information of the time dependence is lost.

Recommended models for combination of time dependent loads in these cases are

a) The Ferry-Borges-Castanheta Load Model.
b) Turkstra’s rule

It should be noted that these models also apply for other time dependent events and not only for loads.

• The Ferry-Borges-Castanheta Load Model, Ferry-Borges (1972).

This model is greatly simplified such that the mathematical problems involved with estimation of a sum of loading processes is avoided.
For each load process in \( \mathbf{X} \) it is assumed that the load changes after equal so-called elementary intervals of time \( \tau_i \). This is illustrated in Figure 6.14, where the reference period \( T \) (e.g., one year) is divided into \( n_i \) intervals of length \( \tau_i = T / n_i \) each, where \( n_i \) denotes the repetition number. Further, it is assumed that the load is constant in each elementary interval. For a load process with marginal distribution \( F_{X_i}(x_i) \), the extreme value distribution in the reference period \( T \) is determined as

\[
F_{\max x_i}(x_i) = (F_{X_i}(x_i))^{n_i}
\]  

(6.32)

![Figure 6.14](image)

Figure 6.14 Estimation of extreme values with elementary intervals.

When combinations of load processes \( X_1, X_2, X_3, \ldots, X_k \) are considered, it is assumed that the loads are stochastically independent with positive integer repetition numbers \( n_i \), where

\[
n_1 \leq \ldots \leq n_i \leq n_k
\]  

(6.33)

and where the number of repetitions to be applied on each of the load processes are

\[
n_i / n_{i-1} \in \mathbb{N}_+
\]  

(6.34)

where \( \mathbb{N}_+ \) is the set of positive natural numbers. The conditions in Eq. (6.34) are illustrated in Figure 6.15 for \( k = 3 \) and \( n_1 = 2, \ n_2 = 6 \), and \( n_3 = 12 \). The method is then to use the extreme value distribution of \( n_i / n_2 \) cycles of \( X_1(t) \) in combination with \( X_2(t) \).
The combined distribution may be raised to the power \( n_2 / n_3 \) and to be used together with \( X_3 \), and so on.

\[
Y_1 = \max\{X_1(t) + X_2(t^*) + \ldots + X_k(t^*)\}
\]

\[
Y_2 = X_1(t^*) + \max\{X_2(t) + \ldots + X_k(t^*)\}
\]

**Figure 6.15** Definition of time intervals and repetition numbers for the Ferry-Borges-Castanheta load model.

Although the Ferry-Borges-Castanheta load model presented above is a gross simplification of the real loading situation, experience indicate that the model is capable of reflecting the most important characteristics of load combination.

- Turkstra’s rule

This rule is an approximation to determine the largest maximum of a sum of \( k \) loads or load effects. Using Turkstra's rule, Turkstra (1970), the extreme value in the reference period \( T \) is found by considering the following \( k \) stochastic variables

\[
Y_i = \max\{X_1(t) + X_2(t^*) + \ldots + X_k(t^*)\}
\]

\[
Y_2 = X_1(t^*) + \max\{X_2(t) + \ldots + X_k(t^*)\}
\]
where \( t^* \) is an arbitrary point in time. The value \( \max\left\{ X_i(t) \right\} \) may be determined in manner similar to the extreme value evaluation in Eq. (6.27) (or Eq. (6.9)). This is a simple method for combination of loads and is correct for independent \( X_i \)'s. The largest load is determined as the largest of the \( k Y_i \)'s. If the \( Y_i \)'s represent different load effects, all combinations should be checked according to the corresponding failure criteria.

By this rule the reliability of a structure is only checked at those points in time when the *individual* load processes reach their maximum values. Therefore, the reliability will be overestimated. However, it has been shown that this overestimation is usually small, see Turkstra and Madsen (1980).

A conservative approach would be to combine the extremes of several environmental processes or events, assuming that these occur simultaneously.
References


Battjes, A.J. (1972), "Long-Term Wave Height Distributions at Seven Stations around the British Isles", Deutsche Hydrographische Zeitschrift, 25.


Det Norske Veritas (1991), Environmental Conditions and Environmental Loads, Classification Note 30.5.


Ferry-Borges, J., and M. Castanheta (1972), Structural Safety, Laboratorio Nacional de Engenharia Civil, Lisbon, Portugal.


Nordenstrøm, N. (1973), "A Method to Predict Long-Term Distributions of Waves and Wave-Induced Motions and loads on Ships and Other Floating Structures", Classification and Registry of Shipping, Publ. No. 81.


Other Useful References

Wirsching, P.H. (1983), "Probability Based Fatigue Design Criteria for Offshore Structures", document prepared for API.

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