Reliability Based Code for Scuffing of Marine Gear Transmissions.

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Abstract

New machinery designs, new materials and new areas of application call for constant updating and calibration of design and classification Rules. In this paper an attempt is made to use methods developed within design rule development for load bearing structures for this purpose. The developed theory and methods, usually referred to as structures reliability theory, have made considerable advance during the last decade and are used both directly for design analysis and for code calibration. In the paper we show that the methods are equally applicable for development of design rules for marine propulsion systems. This is demonstrated through the calibration of a new method for the scuffing limit state in marine cylindrical- and bevel gears.

INTRODUCTION

In 1992 Det Norske Veritas, DNV, initiated a research project to develop reliability-based calculation models for components in marine propulsion systems. The idea was to use structural reliability methods, which has developed considerably during the last decade mainly for use of analysing of load carrying structures, to improve design rules and remove the causes of the problems, rather than the traditional risk-analysis approach which is more focused on controlling the consequences. The most acute problem at the time the project was initiated was the scuffing in some marine gears recently installed. Scuffing occurs when lubrication fails because of too high contact pressure and sliding causing high temperature, or if the lubrication oil is shuffled away. In a number of cases, gears have had to be replaced, causing delays and economic loss for the shipowner and problems for the gear manufacturer. It was this which led us to start analysing and modelling potential failure modes in cylindrical and bevel gears. Hazard identification suggested we model fatigue, surface durability and scuffing. The conclusion, following modelling and analysis of the fatigue and surface durability problems, was that the calculation methods give sufficient reliability for the first two potential failure mechanisms. Such problems occurred very seldom, and the calculated small probabilities were in good agreement with this reality. For the scuffing problem the situation was different. Some of the calculated probabilities were very high, a fact confirmed by experience. Scuffing problems occurred far too frequently, therefore the calculation method had to be revised.

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A traditional analysis is deterministic, and a design is checked by verifying that calculated strength is not exceeded by a characteristic load/stress. To account for the uncertainties in the input data and the computational models, conservative choices for parameters are usually employed in the analysis, e.g. 1 percentiles for material parameters, 99 percentiles for loads. In addition a safety factor is used, such that the calculated characteristic loads/stresses multiplied by the safety factor should be smaller than the characteristic strength multiplied by the safety factor.

Since the number of uncertain variables is large and the parameters may be correlated, the result of a deterministic calculation procedure leads to unknown reliabilities, unless it has been calibrated by reliability methods, which was the approach chosen. For the designer the changes will be small, since a deterministic rule is still used for design approval.

In a probabilistic analysis, uncertain parameters are modelled by distributions, and the failure probability is calculated as the load's (working stress) probability of exceeding the strength. In addition the reliability analysis, as implemented in the general purpose analysis program PROBAN gives sensitivity and importance factors. The importance factors tell how much of the uncertainty (variance) in the results stem from each parameter. Furthermore, the advantage of the probabilistic approach is that uncertainties in analysis input are quantified. This implied that we had to determine the uncertainty range (distribution) on a number of parameters in the analysis. The analyses provided feedback on the importance of these uncertainties.

The probabilistic analysis also has the advantage that it can be used to calibrate deterministic design rules with a selected accepted target reliability, see [1,2]. Choice of target reliabilities is still a controversial issue in many organisations. Some choose technology-based criteria for acceptance, like the As Low As Reasonable Practicable (ALARP) criterion, while DNV has chosen rule-based criteria [3].

It turns out, however, that the choice of target reliabilities based on different principles gives similar results. Such principles are:

- Target reliabilities should be calibrated against well established engineering practice.
- Comparison with similar designs where experience is transferable.
- Based on decision theoretical analysis.
- Based on tabulated values, including considerations of redundancy, warning time, and the consequences of failure.

For the scuffing problem, deciding a target reliability was particularly simple, since scuffing is primarily an economic problem. The target reliability level was therefore chosen based on a cost benefit analysis.

**RELIABILITY METHOD**

The reliability analysis is carried out by a First-Order Reliability Method (FORM). These methods are well established and for general references; see [4,5].

A general purpose reliability program has been applied in the analysis, [4]. In the reliability analysis, failure is defined through the limit state function \( g(z) \), which is negative when failure occurs. \( z \) is the vector of basic variables, i.e. load, resistance, stiffness, geometry, statistical and model uncertainty variables. After a transformation of the basic variables into independent and standardised normal variables \( u_i \), the program determines the most probable failure condition (the design point) by an iterative procedure (optimisation). The design point has the coordinates \( u^* \), where:

\[
 u^* = \beta \alpha 
\]

(1)

b is the first-order reliability index, and \( \alpha \) is a unit vector of sensitivity factors. The probability of failure, \( P_f \), and the corresponding reliability index, \( \beta \), are defined and approximated as:
\[ P_F = P(g(z) < 0)) = \int_{g(z) < 0} f(z) \, dz = \Phi(-\beta) \]  
(2)

where \( \Phi() \) is the standardised normal probability integral. The contribution to the total uncertainty from basic variable \( z_i \) is roughly given by the value \( \alpha^2 \) where the variables are independent, and where \( \alpha \) is the \( i \)-th coordinate of the \( \alpha \)-vector when \( z_i \) is independent of the other basic variables. If the limit state is approximated by a second order surface at the design point the analysis method is called SORM. For this and other more accurate methods the generalised reliability index is defined:

\[ \beta_R = \Phi^{-1}(P_F) = \beta \]  
(3)

The theory is easily generalised to union of events, intersection of events and union of intersection of events where each component is defined as:

\[ z \in \{(g(z) < 0)\} \]  
(4)

**MECHANICAL MODEL**

Scuffing is defined as micro welding on the flank surface of the teeth on a pair of meshing toothed wheels. A semi-empiric model [6], which is used for calculation of the scuffing load and the scuffing capacity is described. This model can be formulated as a limit state and used for probabilistic analyses of the flank surfaces and reliability based calibration of the design rules. The scuffing load capacity model is expressed as the ratio between two different temperatures, given as:

\[ S_s = \frac{\vartheta_s - \vartheta_{oil}}{\vartheta_B - \vartheta_{oil}} \]  
(5)

where \( S_s \) is the scuffing safety factor, \( \vartheta_s \) is the scuffing temperature, \( \vartheta_B \) is the maximum contact temperature along the path of contact and \( \vartheta_{oil} \) is the oil temperature before it reaches the mesh. (The DNVC rules of July 1993 requires that in every point along the path of contact \( S_s > 1.5 \) for propulsion gears and \( S_s > 1.4 \) for auxiliary gears. In addition a minimum temperature difference is required, given as \( \vartheta_B \leq \vartheta_s - 50^\circ C \)). The limit state function may be formulated as the difference between the scuffing capacity temperature and the scuffing load temperature:

\[ G(z) = (\vartheta_s - \vartheta_{oil}) - (\vartheta_B - \vartheta_{oil}) = \vartheta_s - \vartheta_B \]  
(6)

Hence the failure probability, i.e. the probability of scuffing, is equal to the probability of the scuffing temperature becomes less or equal to the maximum contact temperature along the path of contact, equations (2) and (6). This corresponds to \( S_s = 1.0 \) in the deterministic model, equation (5).

The complete calculation model, which is given in [6], shows that both \( \vartheta_s \) and \( \vartheta_B \) are functions of several basic variables. These basic variables are related to the lubrication oil conditions, the geometry of the gear wheel- and tooth designs, the material properties, manufacturing accuracy and the operational conditions. These basic variables which are necessary to do a complete scuffing analysis may be probabilistic.

One of these basic variables is the tip relief on the pinion and on the wheel. The tip relief is found to be a stochastic variable with a bimodal nature, i.e. a two peaked distribution where each peak has equal probability content. This two peaked distribution is simplified by introducing two independent beta distributions, one lower- \( C_{ul}\) and one upper distribution \( C_{u}\). Thus the actual scuffing probability, \( P(\text{Scuffing}) \), is equal to half of the calculated probability, where the calculated probability is modelled as series system of two single events, equation (7)
\[ P(\text{Scuffing}) = \frac{P_{\text{Calculated}}(\text{Scuffing})}{2} = \frac{P_{\text{LO}}(\text{Scuffing}) \cup P_{\text{UP}}(\text{Scuffing})}{2} \quad (7) \]

The single events \( P_i(\text{Scuffing}) \), where the index \( i \) is equal to LO or UP and contains the distributions \( C_{aLO} \) and \( C_{aUP} \) respectively, are equal to the failure probabilities given by equations (2) and (6).

The limit state is coded in FORTRAN-77 and linked to the general purpose probabilistic analysis program, which does the probability calculations in this scuffing study by using the First- (FORM) or the Second- Order Reliability Method (SORM).

**UNCERTAINTY MODELLING**

The key topics of probabilistic modelling and probabilistic calculation is to handle uncertainty. Some parameters in the scuffing model are of deterministic (or almost deterministic) nature, e.g. geometry, and they are not modelled as stochastic. Other variables are uncertain parameters and they are modelled as stochastic variables.

The established distributions and applied uncertainties, are based on knowledge, data, experience and reasonable assumptions.

To do a scuffing reliability analysis of a cylindrical- or a bevel gear transmission it is also important to assess the following items:

- Which body is the driven member, the pinion or the wheel.
- Hardening process of the pinion and the wheel (case hardening, shot peened, nitrided, induction or flame hardening, non hardening or cast steel).
- Type of lubrication condition (spray, dip or fully submerged).

These items are all included in this analysis.

**CODE CALIBRATION**

The main idea of code calibration is to use full distribution reliability methods for calibration of deterministic calculation methods. Through a code calibration of a single safety factor code a common safety factor for different designs is calculated, such that all the designs achieve an approximately uniform level of safety, see Figure 1, about a predefined safety level known as the target reliability. If the code is a partial safety factor code, several common safety factors are calculated.

In a code calibration both the limit state, representing the failure mode for probabilistic calculation, and the code, usually equal or similar to the limit state, are modelled, [1,2]. Data describing a selected set of representative designs are defined in the code calibration input module. These designs are meant to represent the scope of the calibrated code. A target reliability is specified and design variables are identified. Design variables are those variables the designer is able to change to improve unreliable systems. Changes in the design may cause increasing expenses, thus it is important to choose the most cost efficient design solutions. A parametric sensitivity study, executed in front of the calibration, may help to select the design variables. In the computer tool described in [1,2] the design variables and the safety factor(s) are manipulated such that all the code function values become zero, and the failure probabilities are as close to target as possible. A design that satisfies the first condition is called a minimum design. Using the calculated, and calibrated, common safety factor(s) in a code, will assure that the final product achieves a failure probability approximately equal to the target reliability. However, it is required that the design of the final product is inside the scope of the calibrated code if the reliability is to be guaranteed.
Figure 1: Different gear designs: Before rule calibration (Data set 1), after calibration with one safety factor and target reliability $10^{-2}$ (Data set 2), partial safety factor and target reliability $10^{-2}$ (Data set 3), one safety factor and target reliability $10^{-3}$ (Data set 4), partial safety factor and target reliability $10^{-3}$ (Data set 5).

RESULTS

Figure 2 and Figure 3 show the calculated average importance factors, i.e. the ‘basic variables’ relative contribution to uncertainty, for cylindrical- and bevel gear transmissions, respectively. The chart to the right is the calculated average for the considered cylindrical gear transmissions and the chart to the left is for the considered bevel gears. Based on these importance factors it can be seen that most of the uncertainties are introduced by the FZG oil class, thus a single safety factor code is proposed after testing various formulations with multiple safety factors. The safety factors, SS, is put into the code where it was found that it will capture most of the uncertainty introduced by the basic variables. A proposed code is given as:

$$\left(\frac{\vartheta_k - \vartheta_{oil}}{S_B}\right) - \left(\vartheta_B - \vartheta_{oil}\right) \geq 0$$

(8)

The scuffing code is calibrated against the target reliability, which in this case is chosen to be equal to $10^{-4}$ failures during the life time, i.e. (if a frequentistic interpretation of the reliability was accepted) it is expected that 1 gear transmission suffers scuffing in a population of 10000 designs within the defined scope during the life time. This may be interpreted as low reliability, however, the consequences for life, environment, property and the cost of increasing the reliability have to be considered when deciding the reliability level. Since scuffing is primarily an economic problem the target reliability level was chosen based on a cost benefit analysis.

By studying the parametric sensitivity in the probability analysis, it is decided to use the tip relief and the FZG oil class as design variables. It is assumed that these design variables are the most cost efficient variables when improving the reliability in a design $\beta_{Target} = 3.72$ process. This is decided based on evaluating / and / where q is a design variable and is given by FORM.
A method to quantify the quality of the code calibration, is the least squared formula, equation (9). This equation uses the reliability index as input, where $\beta_{Target} = 3.72$ corresponds to the chosen reliability level $10^{-4}$.

$$\Delta \beta^{\text{Spread}} = \sqrt{\sum_i \left( \beta_{Target} - \beta_i \right)^2}$$

(9)

where $\beta_i$ is the reliability index which corresponds to the failure probabilities, equation(7), for the different designs. A good code has its $\Delta \beta^{\text{Spread}}$ close to zero. The penalty function, $p_{pen}$, used here to minimise $\Delta \beta^{\text{Spread}}$ is a simple quadratic function:

$$p_{Pen} = \sum_i \left( \beta_{Target} - \beta_i \right)^2$$

(10)

Before calibration the mean probability of failure in the different cylindrical gear designs was $P_F = 6.4 \cdot 10^{-2}$ with a spread in the range from $1.0 \cdot 10^{-10}$ (zero probability) to $2.9 \cdot 10^{-1}$. For bevel gears the mean probability of failure before calibration was $P_F = 8.1 \cdot 10^{-2}$ with a spread in the range from $1.0 \cdot 10^{-2}$ to $5.0 \cdot 10^{-1}$. After calibration the mean probability of failures were $P_F = 1.0 \cdot 10^{-4}$ (the target) for both types of gear transmissions. The ranges were reduced to $8.3 \cdot 10^{-5}$ to $5.1 \cdot 10^{-4}$ for cylindrical gears, and to $7.5 \cdot 10^{-5}$ to $3.9 \cdot 10^{-4}$ for bevel gear designs. The deterministic safety factor giving $P_F = 1.0 \cdot 10^{-4}$ was found to be $S_B = 1.5$ for cylindrical- and bevel gears, see[7].

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**Figure 2**: Importance factors in average for cylindrical gears.
**CONCLUSIONS**

The use of reliability based rule development or direct use of reliability models has been shown to have the potential of increasing or balancing the reliability of important components in propulsion systems and auxiliary machinery. This conclusion is probably also valid for mechanical components in other areas. Probabilistic analyses based on full distributed reliability methods describe the uncertainty and predict the reliability of machinery components more realistically than the deterministic analyses. Unnecessary conservative assumptions or design based on most likely situations can be avoided. The probabilistic scuffing model is verified against experience and is shown to be able to differentiate between reliable and unreliable marine cylindrical- and bevel gear transmissions.

Importance factors identify the stochastic variables that contribute most to the uncertainty. This information tells the designer which variables that are candidates for closer examination. Quantitative measurements, like the importance factors and the sensitivity of the different design parameters (e.g. FZG-class, tip relief, roughness etc.), can be used to identify which design parameters influence the reliability of a machinery component in a propulsion system most. This information can be used by the designer, the operators and the surveyors to improve the design-, operation-, inspection and maintenance routines.

To do cost effective reliability based design the parametric sensitivity is important information, because it immediately shows the effect of changing a design variable or reducing an uncertainty.

Reliability based code calibration of the scuffing model (or any mechanical model) by use of probabilistic methods will assure that future designs satisfy the specified level of safety.

*Figure 3: Importance factors in average for bevel gears.*
REFERENCES


